

# Retarded Gravitation Theory: the Gravitational Velocity Effect in Galactic Rotation Curves, and the Flyby Anomaly

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We reformulate Newtonian gravitation in a Lorentz covariant way. This retarded gravitation theory (RGT) retains the notion of force, making it easy to solve many-body problems with existing techniques. The new gravitational force depends upon both retarded position and velocity; this explains observed galactic rotation curves as due to “velocity drag” from a large number of nearby co-rotating stars. We calculate this velocity effect for a test particle introduced in a state of Newtonian equilibrium, into a model galaxy. The test particle is speeded up. Dark matter may well exist, but it is not needed in this theory. RGT has the refutable local consequence that the earth’s rotation would influence spacecraft trajectories, by approximately the amount of the observed flyby anomaly. We use the new force to calculate the trajectories of spacecraft flying by a simplified model of the earth as a rotating sphere. For the first flyby of Galileo, the calculated velocity increase is about 1.5 times larger than that observed. There are similar discrepancies for Galileo-2, NEAR and Cassini. We expect these discrepancies will be resolved by eliminating various simplifying assumptions. However, RGT primarily assumes only Lorentz covariance, so even its failure would have major implications for present-day physics.

## I. INTRODUCTION

### A. The problem of galactic rotation curves

Galactic rotation curves have been well studied for decades. The luminous matter in a typical galaxy is concentrated at the centre, and thins out towards the edge. If  $M_r$  is the total mass out to (mean) radial distance  $r$  from the centre of the galaxy, then, based on the observed luminous matter,  $M_r$  effectively becomes constant after a certain value of  $r$ . On Newtonian gravitation, the theoretical expectation is that, beyond this value of  $r$ , rotation velocities should decline as  $r^{-\frac{1}{2}}$ . Planetary orbits in the solar system accurately conform to this expectation, but galaxies present a striking anomaly.

For spiral galaxies, it has been consistently found on observation[1–3] that the rotation velocity *increases* with  $r$ , reaching a roughly *constant* value at the edge of the galaxy. The conclusion is that either (a) galaxies consist predominantly of dark matter, or that (b) Newtonian gravitation fails for the galaxy, and must be modified. Current opinion (“standard model of cosmology”, or  $\Lambda$ -CDM model) is strongly in favour of (cold) dark matter, as its name suggests. The strength of opinion in favour of this belief is demonstrated by the vast speculative literature concerning the constitution of this hypothetical dark matter.<sup>1</sup>

Now, it is quite possible that there is much dark matter in the galaxy. However, merely postulating the existence of dark matter does not quite explain galactic rotation curves. Accordingly, it is further postulated that the dark matter is distributed in a peculiar way, like a halo, with its density reaching a peak where the density of luminous matter thins out to zero. Since the existence of this dark matter is inferred solely by its gravitational effects, which (regardless of its composition) are assumed to be identical to those of luminous matter, it is hard to understand why luminous matter and dark matter should be distributed so very differently. The peculiar distribution required of the hypothetical dark matter seems an artificial hypothesis invented just to save the Newtonian theory from manifest failure.

### B. Earlier modifications of Newtonian dynamics

Theories of modified Newtonian dynamics have been proposed earlier. However, Milgrom’s theory[4] is purely phenomenological. It leaves Newtonian gravitation untouched at the level of the solar system, modifying it only for low accelerations and for large distances, on the scale of the galactic radius. Consequently, the theory is untestable at the level of the solar system. A similar acceleration law is obtained using Moffat’s scalar-tensor-vector theory which involves more complex theoretical considerations [5], but still relies on phenomenological fitting of parameters. That is, even if these theories were empirically refuted, that would leave the rest of physics unaffected. Thus, while opinion about the existence of dark matter is still somewhat divided, existing theories on both sides of the divide are unsatisfactory.

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<sup>1</sup> We will not even attempt to review this literature. Some contenders are MACHOs (Massive Compact Halo Objects), WIMPs (Weakly Interacting Massive Particles), and RAMBOs (Robust Associations of Massive Baryonic Objects).

### C. The new approach: general considerations

The aim of this paper is to explain a simple way to resolve this long-standing problem, by proceeding on a sound theoretical analysis, and without accumulating hypotheses one way or another. Our correction to Newton's law of gravitation is obtained not *ad hoc*, but by invoking Lorentz covariance.

To begin with, let us note that that Newton's laws of motion and the law of gravitation come as a package deal,[6] even though they are usually regarded as separate. Newton's laws of motion are not, by themselves, refutable. In fact, the second law, regarded as a *definition* of force, is not even a good definition, since its right hand side is not properly defined because Newtonian physics failed to define a physical measure of "equal intervals of time". However, the *combination* of Newton's laws of motion and the law of gravitation is refutable and leads to physics (after force is eliminated). The two laws, thus, stand or fall together. Consequently, if the laws of motion are modified, as did happen after the advent of special relativity (which remedied the lacuna in Newtonian mechanics by defining a physical measure of equal intervals of time), then the law of gravitation too *must* be modified or replaced.

All this is well-known and general relativity theory (GRT) did modify Newtonian gravitation. However, GRT is not very helpful if one actually wants to solve a billion-body problem, as is required to understand the structure of galaxies. GRT does not admit point masses; nor is it clear *exactly* how, in the absence of a generally covariant statistical mechanics, the properties of the "fluid" in the stress-energy tensor of GRT are to be related to a discrete or particulate distribution of matter. Because of these difficulties, astronomers' expectations about galactic rotation curves still proceed on intuition built around Newtonian gravitation.

Accordingly, this paper proposes a different modification of Newtonian gravitation, which we will call retarded gravitation theory (RGT). Unlike GRT which is generally covariant, RGT modifies Newtonian gravitation to make it Lorentz covariant. Lorentz covariant gravitation is the minimal necessity after special relativity. Also, unlike GRT, and its geodesic hypothesis, RGT retains the notion of force. We emphasize that it is *not* our intention in this paper to ideologically counter GRT or the geodesic hypothesis, by harking back to the notion of force. Rather, we retain force purely as an operational convenience to be able to solve more easily the gravitational many-body problem in various astronomical contexts, using familiar techniques, but improving upon Newtonian gravitation.

The need to modify Newtonian gravitation was noticed long ago by Poincaré [7] although his starting point was different (a theory of the electron), and he explored various possible mathematical expressions for the gravitational 4-force without fixing on a definite expression for it, which he rightly emphasized would be premature in

the absence of empirical data. He obviously never explored the consequences of modified gravitation for the rotation curves of galaxies, or the trajectories of spacecraft, which consequences were not known in his time.

Unlike Newtonian gravitation, where the force acts instantaneously, the gravitational force in RGT, to be Lorentz covariant, is taken to propagate with the speed of light. This means that the basic equations of motion of RGT are functional differential equations (FDEs) instead of the ordinary differential equations (ODEs) of Newtonian gravitation or the partial differential equations (PDEs) of GRT. As this author has emphasized, the use of FDEs has many advantages. For example it leads to a clean resolution of the classical paradoxes of thermodynamics, through new qualitative features, absent in Newtonian mechanics.[8, 9] (In retrospect, those "paradoxes of thermodynamics" are better understood as pointing to the manifest empirical failure of Newtonian physics.) In fact, the electrodynamic many-body problem, too, involves FDEs, and this author has already explained[10] how to solve retarded FDEs in such a physical context. (However, in the special case of the calculation below, we explain how to get by just by solving ODEs.)

### D. The flyby anomaly

One problem with theories of modified gravity has been the inability to test such theories by means of experiments performed here and now. This situation has changed recently. Between 1990 and 2005, six NASA spacecraft flew by earth, using the technique of earth gravity assist, to either gain or lose heliocentric orbital energy. Anderson et al [11] reported that anomalous orbital energy changes were observed. The anomalies were extremely small, corresponding to an unexplained velocity difference of the order of a few mm/s, but these were well beyond systematic experimental error, which ranged from 0.01 mm/s to 1 mm/s. The causes of the anomalies could not be explained despite a careful audit and consideration of various possible factors including general relativistic effects[12] (though it has been suggested that the anomaly may be an artefact of the observation process [13]).

Further, Anderson et al found an empirical formula which fitted all six flybys.

$$\frac{\Delta V_\infty}{V_\infty} = K(\cos \delta_i - \cos \delta_o) \quad (1)$$

where  $\Delta V_\infty$  was the difference between the incoming and outgoing asymptotic velocity in a *geocentric* frame (conceptually the hyperbolic excess velocity at infinity of an osculating Keplerian trajectory, which difference ought to have been zero on the Newtonian theory) and  $\delta_i$  and  $\delta_o$  were the declinations of the incoming and outgoing asymptotic velocity vectors. The constant  $K$  was expressed in terms of the Earth's angular rotational veloc-

ity  $\omega_E$  of  $7.292115 \times 10^{-5}$  rad/s, its mean radius  $R_E$  of 6371 km and the speed of light  $c$  by  $K = \frac{2\omega_E R_E}{c} = 3.099 \times 10^{-6}$ . Combined with the dependence on declination in (1), this suggested that the observed anomalous effect in the flybys depended systematically on the rotation of the earth.

Newtonian gravitation *cannot* account for any such effect due to the earth's rotation, which is expected on RGT.

## II. THE RETARDED GRAVITATIONAL FORCE

To derive the new force of RGT, we start with a reference frame in which the test particle (“attracted body”) is a mass point located at the origin. The gravitational interaction is not instantaneous but travels at the speed of light, and for the purposes of this paper we take it to be retarded. Accordingly, the “attracting body” is located at the retarded position described by the 4-vector  $X = (ct, \vec{x})$  and moving with a 4-velocity  $V = \gamma_v(c, \vec{v})$ , both at *retarded* time  $t = -\frac{r}{c}$ . Here,  $\vec{x} = (x, y, z)$ ,  $r = \sqrt{x^2 + y^2 + z^2}$ , and  $\gamma_v = (1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$  is the Lorentz factor.

Let  $F = (T, \vec{f})$  be the 4-force experienced by the attracted body. This 4-vector transforms in the same way as the 4-vectors  $X$  and  $V$ , so we take it to be given by a linear combination

$$F = aX + bV, \quad (2)$$

where  $a$ , and  $b$  are Lorentz invariants to be determined. Since  $a$  and  $b$  are Lorentz *invariant*, the expression (2) for the 4-force  $F$  would be Lorentz covariant, as required.

For the case where the attracting body is at rest ( $\vec{v} = 0$ ), we require that the 3-force must approximately agree with the Newtonian gravitational force  $\vec{f} = k(\frac{x}{r^3}, \frac{y}{r^3}, \frac{z}{r^3})$ , where  $k = Gm_0m_1$ , the two (rest) masses are  $m_0$  and  $m_1$ , and  $G$  is the Newtonian gravitational constant. (Note that the sign conventions we are using are the opposite of the usual ones, since the “attracting body” is at  $X$ , and the force is in the direction of its retarded position.) Therefore,  $a \approx \frac{k}{r^3}$ . This suggests that  $a = -\frac{kc^3}{a_1^3}$  where  $a_1$  is the Lorentz invariant quantity  $a_1 = X.V = \gamma_v(c^2t - \vec{x}.\vec{v})$ , which equals  $-cr$  when  $\vec{v} = 0$ , and approximately equals  $-cr$  when  $v = \|\vec{v}\|$  is small compared to  $c$ . That is,

$$a = -\frac{kc^3}{(X.V)^3} \approx \frac{k}{r^3} \quad (3)$$

We now use the fact that the components of the 4-force are not independent, but must satisfy the equation[14]

$$F.U = 0, \quad (4)$$

where  $U = \gamma_u(c, \vec{u})$  is the 4-velocity of the particle on which the force acts. This comes about simply since the revised form of the equations of motion is now

$$m_0 \frac{d^2 Y}{ds^2} = F, \quad (5)$$

where  $m_0$  is the rest mass and  $s$  is proper time along the world line of the “attracted particle”,  $Y(s)$ . Since the 4-force  $F$  is parallel to the 4-acceleration of the particle on which it acts, it must be perpendicular to its 4-velocity  $U$  (which is a vector of constant norm). Accordingly, taking the dot product of  $U$  with (2), we obtain

$$0 = a(X.U) + b(V.U) \quad (6)$$

Now the dot products  $X.U$  and  $V.U$  are scalars, or Lorentz invariant, and the Lorentz invariant  $a$  is already determined. Hence, (6) determines  $b$  as a Lorentz invariant. Explicitly,

$$b = -\frac{a(X.U)}{(V.U)} \approx \frac{k}{cr^2} \quad (7)$$

Note that we would not have been able to satisfy the requirement (4) had we already set  $b = 0$  to begin with. This shows that the Lorentz covariant force we seek *cannot* be purely position dependent. Substituting these values of  $a$  and  $b$  in (2), the force in RGT is explicitly given by

$$F = -\frac{kc^3}{(X.V)^3}X + \frac{kc^3}{(X.V)^3} \frac{(X.U)}{(V.U)}V \quad (8)$$

Since the equations of motion (5), and the expression for the force (8) are Lorentz covariant, we can use these expressions in any Galilean frame, and are not tied to any special frame. Note, however, that RGT, unlike GRT, is restricted to Galilean frames.

Now, the rotational velocities of stars in galaxies are in the range of a few hundred km/s, small compared to the speed of light. Accordingly, for this case, in a frame in which the galactic centre is at rest, we can use the approximate expressions for  $a$  and  $b$  given in (3) and (7). This leads to

$$F \approx \frac{k}{r^2} \left( \frac{X}{r} + \frac{V}{c} \right), \quad (9)$$

which simple form exhibits the departure from Newtonian gravitation more clearly for non-relativistic velocities.

The new force has a component in the direction of the velocity of the attracting body. In a spiral galaxy, a typical star has a large number of neighbouring stars rotating in a common direction. Under these circumstances, it is intuitively evident that this velocity component of the new gravitational force will generate a “velocity drag” which will systematically speed up stars. For a typical galaxy, the number of stars is a large number ranging

from  $10^9$  to  $10^{11}$ . And, because of superposition of forces, the velocity-drag effect will typically scale with the number of bodies involved.

The consequences of such a velocity dependence of gravitation should be observable also at the local level of the earth. The “flyby anomaly”, pointed out by Anderson et al, is immediately seen to be of the right order of magnitude ( $\frac{v}{c}$ ) predicted by the velocity-dependent gravitational force of RGT in (9). In fact, the local linear velocity  $v$  of the earth, due to rotation, at declination or geocentric latitude  $\delta$ , is just  $\omega_E R_E \cos \delta$ , so that  $K \cos \delta = \frac{2v}{c}$  in the notation of (1). This already suggests that the empirical formula of Anderson et al for the “flyby anomaly” might have a simple explanation in RGT.

Of course, we need to check things out by solving the equations of motion (especially for spacecraft) using the new force law. If these detailed calculations bear out the above estimates, then the RGT explanation is to be preferred over other theories since RGT makes no hypothesis, assuming only Lorentz covariance, and explains also galactic rotation curves without assuming dark matter or its peculiar distribution.

### III. RESULTS

#### A. Simplifying the FDEs to ODEs

To check out the intuitive expectations (regarding both galaxy and spacecraft) we need to calculate the actual “velocity drag” by solving an  $n$ -body problem. As pointed out above, this involves the solution of a system of FDEs, for which we need to prescribe the *past history* of the particles. A general procedure for doing this in physics was already laid down in an earlier paper by this author,[10] and a similar procedure can be followed in this case, if needed. (The case of stiff (retarded) FDEs is taken up in [15].)

However, for our immediate purposes of estimating the magnitude and consequences of the “velocity drag”, we have an easy way out. Consider a model of a galaxy consisting of  $n$  particles rotating with an elevated velocity larger than would obtain with Newtonian gravitation. We now introduce an  $(n+1)$ st body or a test particle. We suppose that it was initially moving with the velocity prescribed by Newtonian gravitation, and below the elevated velocity of its neighbours. We need to calculate how much the velocity of this  $(n+1)$ st body or test particle will increase as a consequence of the velocity dependence of the gravitational force.

In the scenario sketched above, our real point of interest is the motion of the test particle, or the  $(n+1)$ st body, and specifically in ascertaining the consequences on its motion of velocity-dependent gravitational effects. For this purpose, as a first approximation, it is reasonable to suppose that the test particle has a negligible effect on the motion of the other  $n$  bodies (namely the

rest of the galaxy). The situation is similar in the case of flyby anomaly where our interest is in the spacecraft trajectory and not its tiny effect on the motion of the earth.

In that case, instead of prescribing just the past history of the  $n$  bodies, as required for the solution of FDEs, we may regard the *entire* world lines of the  $n$  bodies as given. In this situation, where the motion of all other particles is fixed, the equation of motion for the  $(n+1)$ st body becomes just an ordinary differential equation. Using the approximation (9), and neglecting the  $\gamma$  factors (for this case), the equations can be rewritten in 3-vector form as

$$\vec{u} = \frac{d\vec{y}}{dt} \quad (10)$$

$$\frac{d\vec{u}}{dt} = \sum_{i=1}^n \frac{Gm_i}{r_i^2} \left( \frac{\vec{x}_i}{r_i} + \frac{\vec{v}_i}{c} \right). \quad (11)$$

Here  $\vec{y}$  is the position vector of the test particle,  $\vec{x}_i$  is the retarded relative displacement vector of the  $i$ th attracting particle (so its absolute retarded position is  $\vec{z}_i = \vec{y} + \vec{x}_i$ ). Thus,  $(ct_i, \vec{z}_i)$  is the point where the backward null cone from  $(ct, \vec{y})$  meets the world line of the  $i$ th attracting particle. The sum on the right hand side of (11) is over all  $n$  attracting masses  $m_i$ , using their retarded distances  $r_i$ , and velocities  $\vec{v}_i$  at the retarded times  $t_i$ .

We numerically solve this problem, using Hairer et al.’s DOPRI program for the solution of ordinary differential equations (since the problem seems only mildly stiff). This code is tried and trusted, despite its limitations.[16]

Since the immediate aim of this calculation is only to estimate the “velocity drag”, we make further simplifying assumptions: we assume symmetric models for both galaxy and earth. Further, given this symmetry, we expect that the sum over retarded quantities will be only slightly different from the sum over instantaneous quantities, which we accordingly use. The results are shown below.

#### B. Units and model for the galaxy

As already explained earlier[10] to minimize computational error, the units must be so chosen that the numbers involved are neither too small nor too large. We solved the equation (11) using as units of length 1 kpc =  $3.08568025 \times 10^{19}$  m, time  $10^{14}$  s  $\approx 10$  million years, mass  $1.98892 \times 10^{35}$  kg  $\approx 10^5$  solar masses. In these units, we have the speed of light  $c = 9.715603488080141 \times 10^2$ , and  $G = 4.5182366574577775 \times 10^{-6}$ .

For our model galaxy, we assumed a large central mass ( $1.5 \times 10^5$  mass units), surrounded by a ring of radius 12 kpc, consisting of 10000 particles, all of 1 mass unit each, rotating with a constant angular velocity which is twice what one would expect on Newtonian considerations. Namely,  $\omega_{\text{ring}} = 3.96 \times 10^{-2}$ , corresponding to a linear velocity of around 146.6 km/s.

We now introduce a test particle at the position 12.2 kpc. We suppose the test particle is initially rotating with an angular velocity expected on Newtonian considerations (taking into account both central mass and mass of the ring), namely,  $\omega_{\text{particle}} \approx 1.94 \times 10^{-2}$  corresponding to a linear velocity of around 72.9 km/s. Since we neglect the effect of the test particle on the other particles, the mass of the test particle is irrelevant, and does not appear in the equations (11).

### C. The solution for the galaxy

The solution is shown in the graph below (Fig. 1). The velocity of the test particle increases so much that it escapes. This is an example of the kind of behaviour possible with the new force law, but impossible with Newtonian gravitation (even with the hypothesis of dark matter).

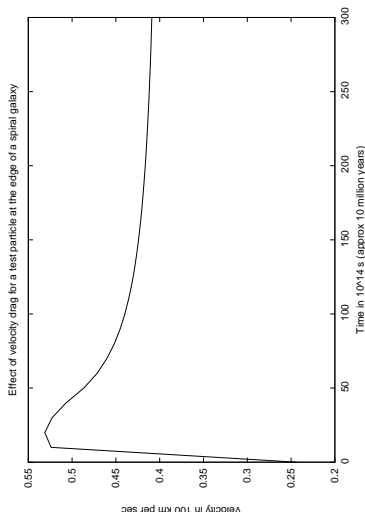


FIG. 1: **The velocity effect of gravitation.** The velocity of a test particle in Newtonian equilibrium in our model galaxy increases. The plot is time vs (scalar) velocity. The time units are approximately 10 million years, while length units are 1 kpc. In this model, a central mass of  $1.5 \times 10^{10}$  solar masses is surrounded by 10,000 particles (each of  $10^5$  solar masses) distributed evenly in a ring of radius 12 kpc rotating with an elevated velocity. The test particle is initially introduced outside the ring at 12.2 kpc.

The solution shown in Fig. 1 brings out that the velocity effect is most pronounced *inside* the ring where the Newtonian force due to the ring vanishes, on account of symmetry, but its velocity-dependent force persists. The sharpness of the velocity increase in the above figure is due to this: modelling the ring as infinitesimally thin, makes the transition from one regime to another abrupt. The velocity of the test particle decays at infinity as expected on Newtonian considerations but remains

persistently larger, and the difference asymptotically approaches a constant.

Thus, we have demonstrated that given a large number of rotating particles, a slowly rotating particle can get substantially speeded up (and even escape) as consequence of the velocity dependent force of RGT. In a spiral galaxy which does have a large number of stars rotating in a common direction, a higher-than-Newtonian rotational velocity is expected on this theory. This velocity drag provides a simple way to understand the “excess” velocity in the rotation curves of spiral galaxies without the hypothesis of dark matter. A roughly constant velocity of rotation, as achieved at the edge of the galaxy, is a very plausible equilibrium state with the new force. Given this simple alternative explanation for observed higher rotational velocities in spiral galaxies, we are not obliged to suppose that is the result of invisible dark matter distributed in some peculiar way.

### D. Units and models for spacecraft

For the case of a spacecraft, our computational units are length = km, mass =  $10^{18}$  kg, and time = 100 s. In these units, the spacecraft asymptotic velocities are typically in the range of hundreds of km per hundred seconds, and the effect we are looking for (in the range of mm/s, or 0.1 meter per hundred seconds) translates to a change in the velocity in the 4th place after the decimal point. As this is a first (“proof of concept”) computation, rather than a real-time engineering computation, to further simplify matters, and reduce computational complexity, we used instantaneous distances instead of retarded distances. (Retarded distances could, of course, be used as was done earlier by this author in the electrodynamic case.)

We use a simplified model of the earth as a perfect sphere of radius  $R_E$ , of uniform density, and rotating with a constant angular velocity  $\omega_E$ ; this is discretized as a system of  $n$  point masses whose (rotational) motions are prescribed in advance, where  $n$  is a large number. (Some care is needed in this discretization. If we take a mesh which is uniform in spherical polar coordinates  $r, \theta, \phi$ , then, to ensure a uniform mass density in the  $z$  direction, we must allow the masses at each mesh point to vary with the polar angle, to compensate for the smaller azimuthal circles at higher polar angles.) Doubtless this model can be improved, allowing for oblateness and known gravitational anomalies etc. But, since our immediate interest is only to explain the physics of the gravitational velocity effect in RGT and indicate how it can be calculated, we postpone such refinements.

To solve for the spacecraft motion, we need initial data for the system of six ODEs (11), namely the initial position and velocity vectors for the spacecraft. Ideally, these should be obtained from direct observation. For our purposes, we use the ephemerides provided online by NASA’s HORIZONS web interface

(<http://ssd.jpl.nasa.gov/horizons.cgi>) which supplies the estimated position and velocity vectors in a geocentric inertial frame using the Earth mean-equator and equinox-of-reference-epoch (J2000.0). Ideally we should transform these coordinates to geocentric equatorial inertial coordinates for the epoch-of-date to take into account variations due to the precession of the earth's rotation axis.

Since the anomalous effect is tiny, to bring out the consequences of a velocity-dependent gravitational force, and to diminish the confounding effects of artefacts of the modelling process, and simplifying assumptions, we also solved for the motion of the spacecraft with the same initial data, but using just the Newtonian force.

### E. The solution for spacecraft

The difference of (scalar) velocity for the two solutions (in our chosen units, km per 100 s) is given below for the case of Galileo's first earth flyby (Fig. 2) using HORIZONS initial data from about 8 hours before perigee (the starting point is 1990-Dec-08 12:01:00.0000). The calcu-

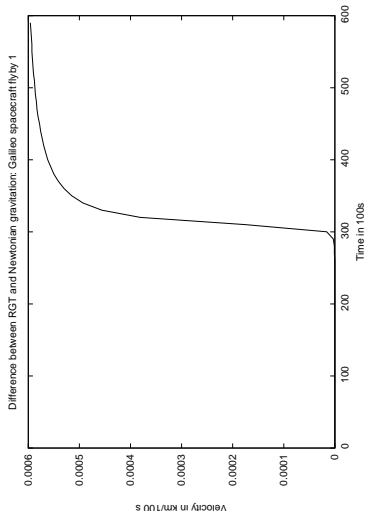


FIG. 2: **Galileo.** The difference in the velocity between the solutions obtained using the new velocity-dependent force and the Newtonian force, for the first flyby of the Galileo spacecraft. The  $x$ -axis is time (in units of 100 s) and the  $y$ -axis is difference of (scalar) velocity in units of km per 100 s.

lated change in velocity shown in the above figure is the right order of magnitude, but about 1.5 times larger than the actual change observed for the Galileo spacecraft.

There is a more serious discrepancy in the results with Galileo's second flyby. In this case, our calculation leads to +7.6 mm/s gain of velocity instead of the value of  $-4.6$  mm/s to  $-8$  mm/s reported by Anderson et al, so the calculated result seems off by a factor of  $-0.5$  to  $-1$ . For the NEAR spacecraft our calculation gives a velocity

gain of only 3.7 mm/s which is too small by a factor of 3.6. Results for the Cassini spacecraft are shown below (Fig. 3). Our calculation yields a change of  $-3.4$  mm/s whereas the change reported by Anderson et al. is  $-2$  mm/s, so the results are again off by a factor of 1.7.

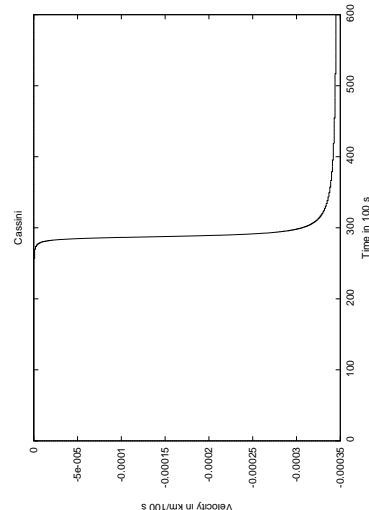


FIG. 3: **Cassini.** The difference in the velocity between the solutions obtained using new velocity-dependent force and the Newtonian force for the earth flyby of the Cassini spacecraft. Same units as before. The calculated change in velocity is  $-3.4$  mm/s compared to the reported change of  $-2$  mm/s.

Given the various simplifying assumptions we have made, the factor of only 1.5 for Galileo-1 is definitely encouraging. The discrepancy in the case of Galileo-2 is larger. However, its significance is not clear. Galileo-2 had a very low perigee, and it seems uncertain exactly how much the atmospheric drag was. If the discrepancy is indeed due to atmospheric drag that would, of course, invalidate Anderson et al's simple formula. The discrepancy in the case of NEAR is more significant. Now, the NEAR spacecraft entered at a low declination, closer to the equator, and exited at a high declination, closer to the pole. Thus, taking into account the oblateness of the earth, with the new force, might resolve this discrepancy. (Within Newtonian gravitation, the oblateness is known to perturb spacecraft orbits, and we suppose that this oblateness was already taken into account in the NASA computational model.)

## IV. DISCUSSION

### A. Galactic rotation curves

Naturally, it would be better to have more realistic models of the galaxy, and the earth, but realistically solving a billion body problem, for the galaxy, using the full retarded many-body FDEs (instead of just the 1-body

ODEs solved here), requires dedicated supercomputing resources which we lack. Indeed, such computational resources are required even for the solution of the many body problem for the galaxy using only Newtonian gravitation, which computation involves a lower computational complexity. Accordingly, we need to proceed iteratively, and the present calculation is the first step.

### B. Flyby anomaly

Regarding the anomalies in the flybys of spacecraft, the calculations shown in the Fig. 2 and 3 already indicate the refutable consequence that velocity-dependent (“anomalous”) changes in spacecraft velocity take place close to periapse. In future publications we expect to eliminate the various simplifying assumptions made in the above calculations, by using a more refined model of the earth, more accurate initial data, retarded times instead of instantaneous times, possible atmospheric drag, and a stiff solver, etc. Ideally, we should modify the existing NASA computational model by applying the changes proposed above, and check if that better fits observed data.

Such a calculation would enable us to design a spacecraft trajectory which could be used to unambiguously check the existence and amount of gravitational velocity effects, and decisively test RGT. Since we have only assumed Lorentz covariance and retardation (or “causality”, for the purposes of this paper), even the failure of RGT would have major implications for physics (though we regard that as very unlikely). The success of the theory, on the other hand, would effectively resolve the long-standing problem of galactic rotation, and the flyby anomaly.

For the present, the above simplified calculations have resolved what was initially a puzzle: how the same formula for the 4-force can lead to such a huge effect in the case of the galaxy, and such a tiny effect in the case of spacecraft.

## V. CONCLUSIONS

We have modified the inverse square law force of Newtonian gravitation in a Lorentz covariant way, mak-

ing gravitational force velocity dependent. The effects of velocity-dependence can be manifested even at non-relativistic velocities, and explain the qualitative features of both galactic rotation curves and the flyby anomaly. We have solved new equations of motion for a test particle introduced in Newtonian equilibrium into a simplified model galaxy. The velocity of the test particle is boosted due to the “velocity drag”, and it escapes. “Velocity drag” from a large number of co-rotating stars provides an alternative way to understand the high velocities actually observed in the rotation curves of spiral galaxies, without assuming any dark matter.

RGT also predicts a velocity effect for spacecraft flybys of earth, which is the right order-of-magnitude for the observed flyby anomaly. Using a simplified model of the earth, and initial data from NASA HORIZONS, we carried out detailed calculations. For the first flyby of the Galileo spacecraft we obtained a velocity gain which is around 1.5 times more than actually observed. However, there are larger discrepancies by a factor of  $-0.5$  and  $3.6$  respectively for the Galileo 2 flyby and the NEAR spacecraft. These discrepancies may be due to neglect of atmospheric drag and oblateness of the earth respectively. For the case of Cassini, our calculated difference is  $-3.4$  mm/s compared to the reported change of  $-2$  mm/s.

We expect to be able to design a tight empirical test for the theory after eliminating the various simplifying assumptions made in the above calculations.

Apart from those assumptions, used to simplify the detailed calculations, the formulation of RGT assumes only Lorentz covariance and retardation. So, a conclusive test (either way) would have major implications for physics.

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